

Lorentz violation and red shift of gravitational waves in brane-worlds

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Abstract

In this paper we study the speed of gravitational waves in a brane world scenario and show that if the extra dimension is space-like, the speed of the propagation of such waves is greater in the bulk than that on the brane. Therefore, the $4D$ Lorentz invariance is broken in the gravitational sector. A comparison is also made between the red shift of such waves and those of the electromagnetic waves on the brane. Such a comparison is essential for extracting the signature of the extra dimension and thus clarifying the question of maximal velocity of gravitational waves in the bulk.

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I. INTRODUCTION

Considering the overall grounds we may expect that the theory of general relativity and quantum theory can be unified in the form of the standard model of particle physics at the Planck energy scale. However, this unification requires quantizing gravity, which results in very fundamental difficulties. One of these difficulties is related to the energy of the fundamental vacuum state and the other concerns the status of Lorentz invariance. The fuzzy nature of space-time in quantum gravity may lead to violations of this fundamental symmetry. During the last two decades theoretical studies and experimental observation of Lorentz invariance violation have received a lot of attention [16, 18, 19]. One possible consequences of Lorentz invariance violation is energy dependent photon propagation velocities. The energy dependence can be constrained by recording the arrival times of photons of different energies emitted by distant objects at approximately the same time [1, 2, 5]. One feature of Lorentz invariance violation to be considered is that the speed of light differs from that in special relativity. According to gravity theories with Lorentz violation, the speed of graviton or the speed of gravitational wave differ from that in general relativity [see e.g.[21]]. Studying the speed of gravitational wave in a Lorentz violating gravity theory will provide different perspectives on quantum gravitational phenomena.

Of promising theories dealing with gravity in recent years are the brane-world scenarios, offering a phenomenological way to test some of the novel predictions and corrections to general relativity that are implied by the M-theory. In such models, it is usually assumed that c is a universal constant. Alternatively, approaches where the speed of gravitational waves can be different from c in a brane-world context have also been considered, see[11–13]. It should be emphasized that the assumption that the maximal velocity in the bulk coincides with the speed of light on the brane should not be taken for granted. In this regard, theories with two metric tensors have been suggested with the associated two sets of “null cones”, in the bulk and on the brane [14]. This is the manifestation of violation of the bulk Lorentz invariance by the brane solution. In some brane-world scenarios, the space-time globally violates $4D$ Lorentz invariance, which results in apparent violations of Lorentz invariance from the brane observer’s point of view due to bulk gravity effects. These effects are restricted to the gravity sector of the effective theory while the well measured Lorentz invariance of particle physics remains unaffected in these scenarios [3, 7, 9].

In this paper, we focus attention on the Einstein field equations on the brane obtained through the well known SMS procedure [26] and address the question of the speed of propagation of gravitational waves in the bulk as well as on the brane. We find a relation between the maximum velocity in the bulk and the speed of light on the brane. Next, we compare the red shift experienced by gravitational waves traveling in the bulk with that of the electromagnetic waves on the brane and show that they are different. We find the signature of the extra dimension which would provide a possible detection mechanism in gravitational wave experiments, thus clarifying the maximal velocity in the bulk. If it turns out that the extra dimension is space-like, then we expect the Lorentz violation effects manifest themselves in gravitational wave experiments.

II. FIELD EQUATION

In the usual brane-world scenarios the space-time is identified with a singular hypersurface (or 3-brane) embedded in a five-dimensional bulk. Suppose now that the background manifold \bar{v}_4 is isometrically embedded in a pseudo-Riemannian manifold v_5 by the map $\mathcal{Y} : \bar{v}_4 \rightarrow v_5$ such that

$$\mathcal{Y}^A_{,\mu} \mathcal{Y}^B_{,\nu} g_{AB} = \bar{g}_{\mu\nu}, \quad \mathcal{Y}^A_{,\mu} N^B g_{AB} = 0, \quad N^A N^B g_{AB} = \epsilon, \quad (1)$$

where $g_{AB}(\bar{g}_{\mu\nu})$ is the metric of the bulk (brane) space $v_5(\bar{v}_4)$ in arbitrary coordinate, $\mathcal{Y}^A(\mathcal{X}^\mu)$ is the basis of the bulk (brane) and N^A is normal unit vector, orthogonal to the brane. Since N^A is a vector field along the extra dimension, we can introduce N^A as follows

$$N^A = \frac{\delta^A_5}{\phi}, \quad N_A = (0, 0, 0, 0, \epsilon\phi), \quad (2)$$

where ϕ is a scalar field. The perturbation of \bar{v}_4 with respect to a small positive parameter y along the normal unit vector N^A is given by

$$\mathcal{Z}^A(x^\alpha, y) = \mathcal{Y}^A + y\phi(x^\alpha, y)N^A. \quad (3)$$

The integrability conditions for the perturbed geometry are the Gauss and Codazzi equations. The perturbation (3) induces a perturbation on the metric $g_{\mu\nu}$ which can be written as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \mathcal{X}_{\mu\nu}(x^\alpha, y). \quad (4)$$

In particular, the linear perturbation obtained from the expansion in y is

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + y\gamma_{\mu\nu}(x^\alpha). \quad (5)$$

To find the perturbed metric, we follow the same definitions as in the geometry of surfaces. Consider the embedding equations of the perturbed geometry written in the particular Gaussian frame defined by the embedded geometry and the normal unit vector

$$\mathcal{Z}_{,\mu}^A \mathcal{Z}_{,\nu}^B g_{AB} = g_{\mu\nu}, \quad \mathcal{Z}_{,\mu}^A N^B g_{AB} = 0, \quad N^A N^B g_{AB} = \epsilon. \quad (6)$$

Substituting equation (3) in (6), we may express the perturbed metric in the Gaussian frame defined by the embedding as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + 2y\phi(x^\alpha, y)\bar{K}_{\mu\nu} + y^2\phi^2(x^\alpha, y)\bar{g}^{\alpha\beta}\bar{K}_{\mu\alpha}\bar{K}_{\nu\beta}, \quad (7)$$

where $\bar{K}_{\mu\nu}$ is the extrinsic curvature of the original brane and the metric of our space-time is obtained at $y = 0$ ($g_{\mu\nu} = \bar{g}_{\mu\nu}$). Using equations (2), (6) and (7), the metric of the bulk is written as

$$dS^2 = g_{\mu\nu}(x^\alpha, y)dx^\mu dx^\nu + \epsilon\phi^2(x^\alpha, y)dy^2, \quad (8)$$

where we have used the signature $(+ - - - \epsilon)$ everywhere. The five-dimensional Einstein equations contain the first and second derivatives of the metric with respect to the extra coordinate. These can be expressed in terms of geometrical tensors in $4D$. The first partial derivatives can be written in terms of the extrinsic curvature

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_N g_{\mu\nu} = \frac{1}{2\phi}\frac{\partial g_{\mu\nu}}{\partial y}, \quad K_{A5} = 0. \quad (9)$$

The second derivatives can be expressed in terms of the projection ${}^{(5)}C_{\mu 5 \nu 5}$ of the bulk Weyl tensor to $5D$

$$\begin{aligned} {}^{(5)}C_{ABCD} &= {}^{(5)}R_{ABCD} - \frac{2}{3}({}^{(5)}R_{A[C}g_{D]B} - {}^{(5)}R_{B[C}g_{D]A}) \\ &\quad + \frac{1}{6}({}^{(5)}Rg_{A[C}g_{D]B}). \end{aligned} \quad (10)$$

In the absence of off-diagonal terms ($g_{5\mu} = 0$) the dimensional reduction of the five-dimensional equations is particularly simple [22, 23]. Thus, the field equations can be split up into three parts

$$\begin{aligned} {}^{(4)}G_{\mu\nu} &= \frac{2}{3}k_5^2 \left[{}^{(5)}T_{\mu\nu} + \left({}^{(5)}T_5^5 - \frac{1}{4}({}^{(5)}T) \right) g_{\mu\nu} \right] \\ &\quad - \epsilon(K_{\mu\alpha}K_\nu^\alpha - K K_{\mu\nu}) \\ &\quad + \frac{\epsilon}{2}g_{\mu\nu}(K_{\alpha\beta}K^{\alpha\beta} - K^2) - \epsilon E_{\mu\nu}, \end{aligned} \quad (11)$$

$$\phi_{;\mu}^{\mu} = -\epsilon \frac{\partial K}{\partial y} - \phi \left(\epsilon K_{\alpha\beta} K^{\alpha\beta} + {}^{(5)}R_5^5 \right), \quad (12)$$

$$D_{\mu}(K_{\nu}^{\mu} - \delta_{\nu}^{\mu} K) = k_{(5)}^2 \frac{{}^{(5)}T_{5\nu}}{\phi}. \quad (13)$$

In the above expressions, $E_{\mu\nu}$ is the electric part of the Weyl tensor and the covariant derivatives are calculated with respect to $g_{\mu\nu}$, *i.e.*, $Dg_{\mu\nu} = 0$. With the brane-world scenario in mind, for deriving the Einstein field equations on the brane it is assumed that the five-dimensional energy-momentum tensor has the form

$${}^{(5)}T_{AB} = \Lambda_5 g_{AB} + {}^{(5)}T_{AB}^{(brane)}, \quad (14)$$

where Λ_5 is the cosmological constant in the bulk and ${}^{(5)}T_{AB}^{(brane)}$ is the energy-momentum tensor of the matter on the brane with ${}^{(5)}T_{AB}^{(brane)} N^A = 0$ and

$${}^{(5)}T_{AB}^{(brane)} = \delta_A^{\mu} \delta_B^{\nu} \tau_{\mu\nu} \frac{\delta(y)}{\phi}. \quad (15)$$

In the spirit of the brane world scenario, we assume Z_2 symmetry about the brane, considered to be a hypersurface Σ at $y = 0$. Using Z_2 symmetry, the Israel's junction conditions are written as

$$K_{\mu\nu}|_{\Sigma^+} = -K_{\mu\nu}|_{\Sigma^-} = -\frac{\epsilon k_5^2}{2} \left[\tau_{\mu\nu} - \frac{1}{3} g_{\mu\nu} \tau \right]. \quad (16)$$

Then, from equation (13) it follow that

$$(\tau_{\nu}^{\mu})_{;\mu} = 0, \quad (17)$$

showing that the energy-momentum tensor $\tau_{\mu\nu}$ is conserved on the brane and represents the total vacuum plus matter energy-momentum. It is usually separated in two parts,

$$\tau_{\mu\nu} = \sigma g_{\mu\nu} + T_{\mu\nu}, \quad (18)$$

where σ is the tension of the brane in $5D$, which is interpreted as the vacuum energy of the brane world and $T_{\mu\nu}$ represents the energy-momentum tensor of ordinary matter in $4D$. Using equations (16) and (18), we obtain the Einstein field equations with an effective energy-momentum tensor in $4D$ as

$${}^{(4)}G_{\mu\nu} = \Lambda_4 g_{\mu\nu} + 8\pi G T_{\mu\nu} - \epsilon k_5^4 \Pi_{\mu\nu} - \epsilon E_{\mu\nu}, \quad (19)$$

where

$$\Lambda_4 = \frac{1}{2}k_5^2 \left(\Lambda_5 - \epsilon \frac{k_5^2 \sigma^2}{6} \right), \quad 8\pi G = -\epsilon \frac{k_5^4 \sigma}{6}, \quad (20)$$

$$\Pi_{\mu\nu} = \frac{1}{4}T_{\mu\alpha}T_{\nu}^{\alpha} - \frac{1}{12}TT_{\mu\nu} - \frac{1}{8}g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} + \frac{1}{24}g_{\mu\nu}T^2.$$

All these $4D$ quantities have to be evaluated in the limit $y \rightarrow 0^+$. They give a working definition of the fundamental quantities Λ_4 and G and contain higher-dimensional modifications to general relativity.

Also, to obtain the equations of gravitational waves in the bulk, first we assume that the bulk space is empty. Using equation (7), the metric of the perturbed brane can be written as follow

$$g_{\mu\nu} = \eta_{\mu\nu} + \xi K_{\mu\nu}, \quad (21)$$

where ξ is a small parameter. By using the Einstein gauge, the equations of gravitational waves become

$$\square K_{\mu\nu} = 0, \quad (22)$$

since $K_{\mu\nu}$ is related to the energy-momentum tensor on brane ($\tau_{\mu\nu}$) by junction conditions, so we can conclude these waves are generated by ordinary matter on the brane.

III. THE BULK GRAVITY EFFECTS AND LORENTZ VIOLATION

In this section, we want to obtain a relation between the maximal velocity of propagation in bulk and on the brane. Let us start by assuming a perfect fluid configuration on the brane. The energy-momentum tensor is therefore written as

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu}, \quad (23)$$

where \mathbf{u} , ρ and p are the unit velocity, energy density and pressure of the matter fluid respectively. We will also assume a linear isothermal equation of state for the fluid

$$p = \gamma\rho, \quad 0 \leq \gamma \leq 1. \quad (24)$$

The weak energy condition [15] imposes the restriction $\rho \geq 0$. In this paper we deal with non-tilted homogeneous cosmological models on the brane, *i.e.* we are assuming that the

fluid velocity is orthogonal to the hypersurfaces of homogeneity. In standard cosmological models, we can also consider $\phi(x^\alpha, y) = \phi(t) > 0$ [22].

Next, we take the metric for our $4D$ universe as

$$\bar{g}_{\mu\nu} = \text{diag}(c_b^2, -a(t)^2 \Upsilon_{ij}), \quad (25)$$

with coordinates (t, x^i) and the 3-metric Υ_{ij} on the spatial slices of constant time. Now, using the Israel's junction condition, we have

$$\begin{aligned} \bar{K}_{00} &= \frac{\epsilon k_5^2 \bar{g}_{00}}{6} [\sigma - (2 + 3\gamma)\rho], \\ \bar{K}_{ii} &= \frac{\epsilon k_5^2 \bar{g}_{ii}}{3} [\sigma + \rho]. \end{aligned} \quad (26)$$

Substituting the above equations in equation (7), the different $4D$ sections of the bulk in the vicinity of the original brane will have the metric

$$g_{\mu\nu} = \Omega^2 \text{diag}(Dc_b^2, -a(t)^2 \Upsilon_{ij}), \quad (27)$$

where

$$\begin{aligned} \Omega^2 &= \left[1 + \frac{\epsilon k_5^2}{6} y \phi(\sigma + \rho) \right]^2, \\ D &= \left[\frac{6 + \epsilon k_5^2 y \phi(\sigma - (2 + 3\gamma)\rho)}{6 + \epsilon k_5^2 y \phi(\sigma + \rho)} \right]. \end{aligned} \quad (28)$$

From (25), we see that the constant c_b represents the speed of light on the original brane, whereas from (27) the speed of the propagation of gravitational waves on the $4D$ section of the bulk is Dc_b^2 . Now, in a brane world scenario where our universe is identified with a singular hypersurface embedded in an AdS_5 bulk, the extra dimension has to be space-like [22]. Therefore, D is always greater than unity ($D > 1$). This leads to apparent violations of Lorentz invariance from the brane observer's point of view due to bulk gravity effects, since the maximal velocity in the bulk becomes more than the speed of light on the brane. It is worth noting that if the energy density of the matter fluid on the brane is zero, we obtain $D = 1$, implying that the maximal velocity in bulk will be the speed of light on the brane. On the other hand, this issue may lead to a modified dispersion relation for propagating modes of gravitational waves. Thus, we should construct a parameterized dispersion relation that can reproduce a range of known Lorentz violation predictions. Also, we can investigate its effects on how these waves propagate. In this regard, the reader is referred to [20, 25].

An interesting analogy exists between the behavior of gravitational waves propagating into the bulk from the brane and the electromagnetic waves crossing one medium into another with different indexes of refraction. This is a reflection of Fermat's principle where the greater speed achieved by gravitational waves in the bulk is taken advantage of when such waves bend slightly into the bulk with the result that they arrive earlier than electromagnetic waves do, the latter being stuck to the brane. Therefore, gravitational waves traveling faster than light would be a possibility. These faster than light signals, however, do not violate causality since the apparent violation of causality from the brane observer's point of view is due to the fact that the region of causal contact is actually bigger than what one would naively expect from the ordinary propagation of light in an expanding universe, with no closed timelike curves in the $5D$ spacetime that would make the theory inconsistent [10].

IV. THE RED SHIFT OF GRAVITATIONAL WAVES

As the present day observations of distant objects involve red shifted spectra, knowing the behavior of the red shift of different waves is necessary for analyzing data. In this section, we compare the red shift of gravitational waves in the bulk with that of the electromagnetic waves on brane. In doing so we consider the usual FLRW line element on the brane with $a(t)$ as scale factor. Therefore, the red shift of electromagnetic waves on the brane is obtained by the usual formula

$$1 + z = \frac{\lambda}{\lambda_0} = \frac{a(t)}{a(t_0)}, \quad (29)$$

where λ and λ_0 are the detected and emitted wavelengths, respectively. On the other hand, using equations (27), (28), the red shift of gravitational waves on brane is given by

$$1 + z = \frac{\lambda}{\lambda_0} = \sqrt{\frac{D(t_0)}{D(t)}} \frac{a(t)}{a(t_0)}, \quad (30)$$

Now, the ratio of the red shift of the gravitational waves to that of the electromagnetic waves is $\sqrt{\frac{D(t_0)}{D(t)}}$ and thus depends on the function $D(t)$. To study the variation of the ratio with time, it is necessary to know the time variation of ρ and σ . From (17) and (18) it follows that

$$\sigma_{;\nu} + T_{\nu;\mu}^{\mu} = 0. \quad (31)$$

For the perfect fluid (23), this is equivalent to

$$\dot{\rho} + (\rho + p)\Theta = -\dot{\sigma}, \quad (32)$$

$$(\rho + p)a_\nu + (\gamma - 1)\rho_{,\lambda}h_\nu^\lambda = \sigma_{,\lambda}h_\nu^\lambda, \quad (33)$$

where $\Theta = u^\mu_{;\mu}$ is the expansion, $a_\nu = u_{\nu;\lambda}u^\lambda$ is the acceleration and $h_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$ is the projector onto the spatial surface orthogonal to u_μ . In homogeneous cosmological models, equation (33) becomes redundant and only equation (32) is relevant. For the case of the constant vacuum energy σ and equation of state $p = (\gamma - 1)\rho$, it yields the familiar relation between the matter energy density and expansion factor a

$$\rho = \rho_0 \left(\frac{a_0}{a} \right)^{3\gamma}. \quad (34)$$

For the case where the vacuum energy is not constant, we need some additional assumption. We know that the time variation of G is usually written as $(\frac{\dot{G}}{G}) = gH$, where g is a dimensionless parameter. The present observational upper bound is $|g| \leq 0.1$ [22, 23]. In what follows we assume that g is constant. Since $G \sim \sigma$ and $H = \frac{\dot{a}}{a}$, we have $\sigma(a) = f_0 a^g$, where f_0 is a constant of integration. From equation (32) we thus find

$$\dot{\rho} + 3\gamma\rho\frac{\dot{a}}{a} = -f_0 g a^{(g-1)}\dot{a}. \quad (35)$$

First, we consider the case where ρ can be expressed in a , similar to (34), i.e. as a power of a . We therefore find $g = \frac{-3\gamma E}{E+f_0}$, and

$$\rho = E a^{-\left(\frac{3\gamma E}{E+f_0}\right)}, \quad (36)$$

where E is a positive constant. In order to simplify the notation we set $f_0 = F_0 E$ and

$$\frac{\gamma}{1+F_0} = \beta + 1, \quad (37)$$

which gives

$$\beta = \frac{\gamma - F_0 - 1}{1 + F_0}. \quad (38)$$

With this notation we now have

$$\rho = \frac{E}{a^{3(\beta+1)}}, \quad \sigma = \frac{E(\gamma - 1 - \beta)}{(\beta + 1)a^{3(\beta+1)}}. \quad (39)$$

Note that one may take $E = \rho_0 a_0^{3(\beta+1)}$ and that $F_0 \neq 0$ ($\beta \neq \gamma - 1$), otherwise $G = 0$. We also have

$$\frac{\dot{G}}{G} = -3(\beta + 1)H. \quad (40)$$

Before going any further, we should be aware of observational bounds on β . The lower bound comes from the obvious requirement $\frac{d\rho}{da} < 0$, while the upper bound comes from the observation that $|g| \leq 0.1$. Thus,

$$-1 < \beta \leq -0.966, \quad \beta = -0.983 \pm 0.016. \quad (41)$$

The next step would be to find the function $\phi(t)$. In standard cosmological models, we can set $\phi = \phi_0 \left(\frac{a}{a_0}\right)^r$. The physical condition $\tau_0^0 = (\sigma + \rho) > 0$ then puts a lower limit on r , namely $r > 1$ [22]. Therefore, we find $D(t)$ as

$$\begin{aligned} D(t) &= \left[1 - \frac{3\epsilon k_5^2 y \phi_0 \rho_0 (1 + \gamma) \left(\frac{a}{a_0}\right)^{r-3(\beta+1)}}{6 + \epsilon k_5^2 y \phi_0 \rho_0 \left(\frac{\gamma}{\beta+1}\right) \left(\frac{a}{a_0}\right)^{r-3(\beta+1)}} \right] \\ &\simeq \left[1 - \frac{\epsilon}{2} k_5^2 y \phi_0 \rho_0 (1 + \gamma) \left(\frac{a}{a_0}\right)^{r-3(\beta+1)} \right], \end{aligned} \quad (42)$$

and

$$\frac{D(t_0)}{D(t)} = \frac{1 - \frac{\epsilon}{2} k_5^2 y \phi_0 \rho_0 (1 + \gamma)}{1 - \frac{\epsilon}{2} k_5^2 y \phi_0 \rho_0 (1 + \gamma) \left(\frac{a}{a_0}\right)^{r-3(\beta+1)}}. \quad (43)$$

Now, if the extra dimension is space-like, $\sqrt{\frac{D(t_0)}{D(t)}}$ is less than unity and so the red shift due to gravitational waves is smaller than that of the electromagnetic waves. On the other hand, if the extra dimension is time-like, $\sqrt{\frac{D(t_0)}{D(t)}}$ is greater than unity. Then, the red shift of gravitational waves is greater than that of the electromagnetic waves.

Since the red shift of gravitational waves is different from that of the electromagnetic waves, this issue may have effects on the detection of gravitational waves. For instance, through the process of finding a correlation between the small scale CMB polarization fluctuations and the galaxy number density at a given red shift, one can determine the local quadrupole moments of the CMB at that red shift. Then, considering these quadrupoles at different patches on the sky and at different red shifts, one can obtain a map of the quadrupole moments during the reionization era [17]. A small part of this quadrupole pattern can be produced by the tensor modes of fluctuations, *i.e.*, gravitational waves. Using the correlation between galaxy distribution and the CMB polarization anisotropies, we can constrain the strength of the primordial gravitational waves which are really important to physicists, see also [4, 6, 8, 24]. Since the red shift of gravitational waves is different from that of electromagnetic waves, one expects the strength of primordial gravitational waves to be modified.

V. CONCLUSIONS

In this paper we have considered a brane-world scenario where the Einstein field equations on the brane were obtained using the usual SMS formalism. Based on this scenario we then showed that within the framework of the standard cosmological models, the red shift associated with gravitational waves moving through the bulk is not equal to the red shift of electromagnetic waves propagating on the brane. Such a difference could be used, in the event of the detection of gravitational waves, to find the signature of extra dimension. This should clarify the question of the maximal velocity in the bulk space. We also showed that if the extra dimension is space-like, the $4D$ Lorentz invariance in the gravitational sector is broken in the sense of having a propagation speed greater than that of light. Gauge fields will not feel these effects, but gravitational waves are free to propagate into the bulk and they will necessarily feel the effects of the variation of the speed of light along the extra dimension.

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